Bibliotheque de Humanisme et Renaissance | ISSN : 0006-1999
Volume 80, Issue 2, 2020

Probability Density Function of Four Variables Associated with Hyper Geometric Functions

Pandhare Balu Shankarsa
Research Scholar, Shri JJT University (Rajasthan).

Dr. Satyaveer Singh
Assistant Professor, Department of Mathematics, Shri JJT University (Rajasthan).

1.1 INTRODUCTION

In this paper a compressive list of twenty integral is given. Proof of all integrals are similar. In what follows we shall take p,q,r and s to be positive integral of the symbol. X_r and Δ(n,a) stand for the sequence of parameter X_1 , X_2 …..X_r and $\frac{\alpha}{n}$,$\frac{\alpha+1}{n}$ … … … …. . $\frac{\alpha+n-1}{n}$ respectively. Also in all be established here after. Proper condition of convergence of the series involved are assumed.

1.2 INTEGRALS INVOLVING HYPERGEOMETRIC FUNCTIONS OF FOUR VARIABLES

[1] $\int_0^1 u^{\alpha-1}\,(1-u)^{\beta-1}\,F_1$(a,b,1-a;α,u;$\frac{u}{2}$)

${F_1}^4[x(1-u)^n, y(1-u)^n, z(1-u)^n, t(1-u)^n]$ du

=

$$\wedge_1\wedge_2 {F_{20}}^4\begin{bmatrix} a_1, a_1, a_1, a_2, b_1, b_1, b_3, b_3, c_1, c_2, c_3, c_1, \Delta(n,\beta), \Delta(n, \alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a), \Delta\left(\frac{n}{2}, \frac{\alpha+\beta-a}{2}\right), \Delta(\frac{n}{2}, \frac{1-b+\alpha+\beta-a}{2}); \text{x; y; z; t} \end{bmatrix}$$

…………………………………………………………..(1.2.1)

where

${F_1}^4\ [a_1, a_1, a_1, a_2, b_1, b_1, b_3, b_3, c_1, c_2, c_3, c_1; x, y, z, t)$

$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q+r}\,(a_2)_s\,(b_1)_{p+q}(b_2)_{r+s}}{p!q!r!s!(c_1)_{p+s}\,(c_2)_q\,(C_3)_r}\, x^p y^q z^r t^s$

Then the probability density function of (1.2.1) is given by

F(U)=$\frac{u^{\alpha-1}(1-u)^{\beta-1}F_1(X_1){F_1}^4[X_2]}{\wedge_1\wedge_2 {F_1}^4[X_3]}$

= 0 elsewhere

Where X_1 = (a,b,1-a;α,u;$\frac{u}{2}$)

X_2=$[(1-u)^n x, (1-u)^n y, (1-u)^n z, (1-u)^n t]$

$$X_3=\begin{bmatrix} a_1, a_1, a_1, a_2, b_1, b_1, b_3, b_3, c_1, c_2, c_3, c_{1;}, \Delta(n,\beta), \Delta(n, \alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a), \Delta\left(\frac{n}{2}, \frac{\alpha+\beta-a}{2}\right), \Delta(\frac{n}{2}, \frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$$

[2] $\int_0^1 u^{\alpha-1} (1-u)^{\beta-1} F_1(a,b,1-a;\alpha,u;\frac{u}{2})$

$F_2{}^4[x(1-u)^n, y(1-u)^n, z(1-u)^n, t(1-u)^n]$ du

=

$$\wedge_1\wedge_2 F_2{}^4\begin{bmatrix} a_1, a_1, a_1, a_2, b_1, b_1, b_2, b_3, c_1, c_2, c_3, c_{3;}, \Delta(n,\beta), \Delta(n, \alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a), \Delta\left(\frac{n}{2}, \frac{\alpha+\beta-a}{2}\right), \Delta(\frac{n}{2}, \frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$$

…………………………………………………………..(1.2.2)

where

$F_2{}^4\ [a_1, a_1, a_1, a_2, b_1, b_1, b_2, b_3, c_1, c_2, c_3, c_{3;} x, y, z, t)$

$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q+r}\,(a_2)_s\,(b_1)_{p+q}(b_2)_r(b_3)_s}{p!q!r!s!(c_1)_p\,(C_2)_q\,(C_3)_{r+s}}\, x^p y^q z^r t^s$

Then the probability density function of (1.2.2) is given by

F(U)=$\frac{u^{\alpha-1}(1-u)^{\beta-1}F_1(X_1)F_2{}^4[X_2]}{\wedge_1\wedge_2 F_2{}^4[X_4]}$

= 0 elsewhere

Where

$$X_4=\begin{bmatrix} a_1, a_1, a_1, a_2, b_1, b_1, b_2, b_3, c_1, c_2, c_3, c_{3;}, \Delta(n,\beta), \Delta(n, \alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a), \Delta\left(\frac{n}{2}, \frac{\alpha+\beta-a}{2}\right), \Delta(\frac{n}{2}, \frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$$

[3] $\int_0^1 u^{\alpha-1} (1-u)^{\beta-1} F_1(a,b,1-a;\alpha,u;\frac{u}{2})$

$F_3{}^4[x(1-u)^n, y(1-u)^n, z(1-u)^n, t(1-u)^n]$ du

=

$$\wedge_1\wedge_2 F_3{}^4\begin{bmatrix} a_1, a_1, a_1, a_2, b_1, b_2, b_3, b_1, c_1, c_1, c_2, c_{3;}, \Delta(n,\beta), \Delta(n, \alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a), \Delta\left(\frac{n}{2}, \frac{\alpha+\beta-a}{2}\right), \Delta(\frac{n}{2}, \frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$$

…………………………………………………………..(1.2.3)

where

$F_3{}^4\ [a_1, a_1, a_1, a_2, b_1, b_1, b_3, b_1, c_1, c_1, c_2, c_{3;} x, y, z, t)$

$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q+r}\,(a_2)_s\,(b_1)_{p+s}(b_2)_q(b_3)_r}{p!q!r!s!(c_1)_{p+q}\,(C_2)_r\,(C_3)_s}\, x^p y^q z^r t^s$

Then the probability density function of (1.2.3) is given by

F(U)=$\frac{u^{\alpha-1}(1-u)^{\beta-1}F_1(X_1)F_3{}^4[X_2]}{\wedge_1\wedge_2 F_3{}^4[X_4]}$

= 0 elsewhere

where

$X_4=\begin{bmatrix} a_{1},a_{1},a_{1},a_{2},b_{1},b_{2},b_{3},b_{1},c_{1},c_{1},c_{2},c_{3;},\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$ [4] $\int_0^1 u^{\alpha-1}\ (1-u)^{\beta-1}$

F_1(a,b,1-a;α,u;$\frac{u}{2}$)

${F_4}^4[x(1-u)^n, y(1-u)^n, z(1-u)^n, t(1-u)^n]$ du

=

$$\Lambda_1\Lambda_2 {F_{23}}^4 \begin{bmatrix} a_{1},a_{1},a_{1},a_{2},b_{1},b_{1},b_{3},b_{1},c_{1},c_{2},c_{2},c_{3;},\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$$

…………………………………………….........(1.2.4)

where

${F_4}^4\ [a_1, a_1, a_1, a_2, b_1, b_2, b_3, b_1, c_1, c_2, c_2, c_3; x, y, z, t)$

$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q+r}\,(a_2)_s\,(b_1)_{p+s}(b_2)_q(b_3)_r}{p!q!r!s!(c_1)_p\,(c_2)_{q+s}\,(C_3)_s}\, x^p y^q z^r t^s$

Then the probability density function of (1.2.4) is given by

F(U)=$\frac{u^{\alpha-1}(1-u)^{\beta-1}F_1(X_1){F_4}^4[X_2]}{\Lambda_1\Lambda_2{F_4}^4[X_5]}$

= 0 else where

where

$X_5=\begin{bmatrix} a_{1},a_{1},a_{1},a_{2},b_{1},b_{2},b_{3},b_{1},c_{1},c_{2},c_{2},c_{3;},\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$

[5] $\int_0^1 u^{\alpha-1}\ (1-u)^{\beta-1}$ F_1(a,b,1-a;α,u;$\frac{u}{2}$)

${F_5}^4[x(1-u)^n, y(1-u)^n, z(1-u)^n, t(1-u)^n]$ du

=

$$\Lambda_1\Lambda_2 {F_{24}}^4 \begin{bmatrix} a_{1},a_{1},a_{1},a_{2},b_{1},b_{2},b_{3},b_{1},c_{1},c_{2},c_{2},c_{3;},\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$$

……………………………………………….......(1.2.5)

where

${F_5}^4\ [a_1, a_1, a_1, a_2, b_1, b_2, b_3, b_1, c_1, c_2, c_2, c_3; x, y, z, t)$

$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q+r}\,(a_2)_s\,(b_1)_{p+s}(b_2)_q(b_3)_r}{p!q!r!s!(c_1)_p\,(c_2)_{q+r}\,(C_3)_s}\, x^p y^q z^r t^s$

Then the probability density function of (1.2.5) is given by

F(U)=$\frac{u^{\alpha-1}(1-u)^{\beta-1}F_1(X_1){F_5}^4[X_2]}{\Lambda_1\Lambda_2{F_5}^4[X_6]}$

= 0 else where

where

$X_6=\begin{bmatrix} a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{2,}, b_{3,} b_{1,} c_{1,} c_{2,} c_{2,} c_{3;,} \Delta(n,\beta), \Delta(n, \alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a), \Delta\left(\frac{n}{2}, \frac{\alpha+\beta-a}{2}\right), \Delta(\frac{n}{2}, \frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$ [6] $\int_0^1 u^{\alpha-1}\ (1-u)^{\beta-1}$

F_1(a,b,1-a;α,u;$\frac{u}{2}$)

${F_6}^4[x(1-u)^n, y(1-u)^n, z(1-u)^n, t(1-u)^n]$ du

=

$$\Lambda_1\Lambda_2 {F_6}^4 \begin{bmatrix} a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{2,}, b_{3,} b_{1,} c_{1,} c_{2,} c_{3,} c_{3;,} \Delta(n,\beta), \Delta(n, \alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a), \Delta\left(\frac{n}{2}, \frac{\alpha+\beta-a}{2}\right), \Delta(\frac{n}{2}, \frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$$

……………………………………………………………..(1.2.6)

where

${F_6}^4\ [a_1, a_1, a_{1,} a_2, b_1, b_2, b_3, b_1, c_1, c_2, c_{3,} c_{1;} x, y, z, t)$

$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q+r}\,(a_{2,})_s\,(b_{1,})_{p+s}(b_{2,})_q(b_{3}\ )_r}{p!q!r!s!(c_{1,})_{p+s}\,(c_{2,})_q\,(C_{3,})_s}\, x^p y^q z^r t^s$

Then the probability density function of (1.2.6) is given by

F(U)=$\frac{u^{\alpha-1}(1-u)^{\beta-1} F_1(X_1) {F_6}^4[X_2]}{\Lambda_1\Lambda_2 {F_6}^4[X_7]}$

= 0 elsewhere

where

$X_7=\begin{bmatrix} a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{2,}, b_{3,} b_{1,} c_{1,} c_{2,} c_{3,} c_{3;,} \Delta(n,\beta), \Delta(n, \alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a), \Delta\left(\frac{n}{2}, \frac{\alpha+\beta-a}{2}\right), \Delta(\frac{n}{2}, \frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$

[7] $\int_0^1 u^{\alpha-1}\ (1-u)^{\beta-1}\ F_1$(a,b,1-a;α,u;$\frac{u}{2}$)

${F_7}^4[x(1-u)^n, y(1-u)^n, z(1-u)^n, t(1-u)^n]$ du

=

$$\Lambda_1\Lambda_2 {F_7}^4 \begin{bmatrix} a_{1,}, a_{1,}, a_{1,}, a_{2,} b_{1,} b_{2,}, b_{3,} b_{4,} c_{1,} c_{2,} c_{3,} c_{1;,} \Delta(n,\beta), \Delta(n, \alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a), \Delta\left(\frac{n}{2}, \frac{\alpha+\beta-a}{2}\right), \Delta(\frac{n}{2}, \frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$$

…………………………………………………………..(1.2.7)

where

${F_7}^4\ [a_1, a_1, a_{1,} a_2, b_1, b_2, b_3, b_4, c_1, c_2, c_{3,} c_{1;} x, y, z, t)$

$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q+r}\,(a_{2,})_s\,(b_{1,})_p(b_{2,})_q(b_{3,})_r(b_{4,})_s}{p!q!r!s!(c_{1,})_{p+s}\,(c_{2,})_q\,(C_{3,})_s}\, x^p y^q z^r t^s$

Then the probability density function of (1.2.7) is given by

F(U)=$\frac{u^{\alpha-1}(1-u)^{\beta-1} F_1(X_1) {F_7}^4[X_2]}{\Lambda_1\Lambda_2 {F_7}^4[X_8]}$

= 0 elsewhere

where

$$X_8=\begin{bmatrix} a_1,a_1,a_1,a_2,b_1,b_2,b_3,b_4,c_1,c_2,c_3,c_1;,\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2});x;y;z;t \end{bmatrix}$$

[8] $\int_0^1 u^{\alpha-1}(1-u)^{\beta-1} F_1(a,b,1-a;\alpha,u;\frac{u}{2})$

$F_8{}^4[x(1-u)^n,y(1-u)^n,z(1-u)^n,t(1-u)^n]$ du

=

$$\wedge_1\wedge_2 F_8{}^4\begin{bmatrix} a_1,a_1,a_2,a_2,b_1,b_2,b_1,b_2,c_1,c_2,c_1,c_3;,\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2});x;y;z;t \end{bmatrix}$$

…………………………………………………….………..(1.2.8)

where

$F_8{}^4\ [a_1,a_1,a_2,a_2,b_1,b_2,b_1,b_2,c_1,c_2,c_1,c_3;x,y,z,t)$

$=\sum_{p,q,r,s=0}^{\infty}\frac{(a_1)_{p+q}(a_2)_{r+s}(b_1)_{p+r}(b_2)_{q+s}}{p!q!r!s!(c_1)_{p+r}(C_2)_q(C_3)_s}x^p y^q z^r t^s$

Then the probability density function of (1.2.8) is given by

$F(U)=\frac{u^{\alpha-1}(1-u)^{\beta-1}F_1(X_1)F_8{}^4[X_2]}{\wedge_1\wedge_2 F_8{}^4[X_9]}$

= 0 elsewhere

where

$$X_9=\begin{bmatrix} a_1,a_1,a_2,a_2,b_1,b_2,b_1,b_2,c_1,c_2,c_1,c_3;,\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2});x;y;z;t \end{bmatrix}$$

[9] $\int_0^1 u^{\alpha-1}(1-u)^{\beta-1} F_1(a,b,1-a;\alpha,u;\frac{u}{2})$

$F_9{}^4[x(1-u)^n,y(1-u)^n,z(1-u)^n,t(1-u)^n]$ du

=

$$\wedge_1\wedge_2 F_9{}^4\begin{bmatrix} a_1,a_2,a_1,a_2,b_1,b_2,b_1,b_2,c_1,c_1,c_2,c_3;,\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2});x;y;z;t \end{bmatrix}$$

…………………………………………………………..(1.2.9)

where

$F_9{}^4\ [a_1,a_2,a_1,a_2,b_1,b_2,b_1,b_2,c_1,c_1,c_2,c_3;x,y,z,t)$

$=\sum_{p,q,r,s=0}^{\infty}\frac{(a_1)_{p+r}(a_2)_{q+s}(b_1)_{p+r}(b_2)_{q+s}}{p!q!r!s!(c_1)_{p+q}(C_2)_r(C_3)_s}x^p y^q z^r t^s$

Then the probability density function of (1.2.9) is given by

$F(U)=\frac{u^{\alpha-1}(1-u)^{\beta-1}F_1(X_1)F_9{}^4[X_2]}{\wedge_1\wedge_2 F_9{}^4[X_{10}]}$

= 0 elsewhere

where

$$X_{10}=\begin{bmatrix} a_{1},a_{2},a_{1},a_{2},b_{1},b_{2},b_{1},b_{2},c_{1},c_{1},c_{2},c_{3;},\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2});x;y;z;t \end{bmatrix}$$

[10] $\int_0^1 u^{\alpha-1}(1-u)^{\beta-1} F_1(\text{a,b,1-a};\alpha,\text{u};\frac{u}{2})$

$F_{10}{}^{4}[x(1-u)^n, y(1-u)^n, z(1-u)^n, t(1-u)^n]$ du

=

$$\Lambda_1\Lambda_2 F_{10}{}^{4}\begin{bmatrix} a_{1},a_{1},a_{2},a_{2},b_{1},b_{2},b_{1},b_{2},c_{1},c_{2},c_{3},c_{1;},\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2});x;y;z;t \end{bmatrix}$$

………………………………………………...………..(1.2.10)

where

$F_{10}{}^{4}\,[a_1,a_1,a_{2,}a_2,b_1,b_2,b_1,b_2,c_1,c_2,c_{3,}c_{1;}x,y,z,t)$

$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q}(a_2)_{r+s}(b_1)_{p+r}(b_2)_{q+s}}{p!q!r!s!(c_1)_{p+r}(c_2)_q(C_3)_s} x^p y^q z^r t^s$

Then the probability density function of (1.2.29) is given by

F(U)=$\frac{u^{\alpha-1}(1-u)^{\beta-1}F_1(X_1)F_{10}{}^{4}[X_2]}{\Lambda_1\Lambda_2 F_{10}{}^{4}[X_{11}]}$

= 0 elsewhere

where

$$X_{11}=\begin{bmatrix} a_{1},a_{1},a_{2},a_{2},b_{1},b_{2},b_{1},b_{2},c_{1},c_{2},c_{3},c_{1;},\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2});x;y;z;t \end{bmatrix}$$

[11] $\int_0^1 u^{\alpha-1}(1-u)^{\beta-1} F_1(\text{a,b,1-a};\alpha,\text{u};\frac{u}{2})$

$F_{11}{}^{4}[x(1-u)^n, y(1-u)^n, z(1-u)^n, t(1-u)^n]$ du

=

$$\Lambda_1\Lambda_2 F_{11}{}^{4}\begin{bmatrix} a_{1},a_{1},a_{2},a_{2},b_{1},b_{2},b_{3},b_{1},c_{1},c_{1},c_{2},c_{3;},\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2});x;y;z;t \end{bmatrix}$$

……………………………………………………...…..(1.2.11)

where

$F_{11}{}^{4}\,[a_1,a_1,a_{2,}a_2,b_1,b_1,b_2,b_3,c_1,c_2,c_{1,}c_{3;}x,y,z,t)$

$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q}(a_2)_{r+s}(b_1)_{p+q}(b_2)_r(b_3)_s}{p!q!r!s!(c_1)_{p+r}(C_2)_q(C_3)_s} x^p y^q z^r t^s$

Then the probability density function of (1.2.11) is given by

F(U)=$\frac{u^{\alpha-1}(1-u)^{\beta-1}F_1(X_1)F_{11}{}^{4}[X_2]}{\Lambda_1\Lambda_2 F_{11}{}^{4}[X_{12}]}$

= 0 elsewhere

where

$$X_{12}=\begin{bmatrix} a_1, a_1, a_2, a_2, b_1, b_2, b_3, b_1, c_1, c_1, c_2, c_3; \Delta(n,\beta), \Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a), \Delta\left(\frac{n}{2}, \frac{\alpha+\beta-a}{2}\right), \Delta(\frac{n}{2}, \frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$$

[12] $\int_0^1 u^{\alpha-1}(1-u)^{\beta-1} F_1$(a,b,1-a;α,u;$\frac{u}{2}$)

$F_{12}{}^4[x(1-u)^n, y(1-u)^n, z(1-u)^n, t(1-u)^n]$ du

=

$$\Lambda_1\Lambda_2 F_{12}{}^4 \begin{bmatrix} a_1, a_1, a_2, a_2, b_1, b_2, b_3, b_1, c_1, c_1, c_2, c_3; \Delta(n,\beta), \Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a), \Delta\left(\frac{n}{2}, \frac{\alpha+\beta-a}{2}\right), \Delta(\frac{n}{2}, \frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$$

……………………………………………………….…..(1.2.12)

where

$F_{12}{}^4\ [a_1, a_1, a_2, a_2, b_1, b_2, b_3, b_1, c_1, c_1, c_2, c_3; x, y, z, t)$

$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q}(a_2)_{r+s}(b_1)_{p+r}(b_2)_{r+s}}{p!q!r!s!(c_1)_{p+r}(C_2)_r(C_3)_s} x^p y^q z^r t^s$

Then the probability density function of (1.2.12) is given by

F(U)=$\frac{u^{\alpha-1}(1-u)^{\beta-1}F_1(X_1)F_{12}{}^4[X_2]}{\Lambda_1\Lambda_2 F_{12}{}^4[X_{13}]}$

= 0 elsewhere

where

$$X_{13}=\begin{bmatrix} a_1, a_1, a_2, a_2, b_1, b_2, b_3, b_1, c_1, c_1, c_2, c_3; \Delta(n,\beta), \Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a), \Delta\left(\frac{n}{2}, \frac{\alpha+\beta-a}{2}\right), \Delta(\frac{n}{2}, \frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$$

[13] $\int_0^1 u^{\alpha-1}(1-u)^{\beta-1} F_1$(a,b,1-a;α,u;$\frac{u}{2}$)

$F_{13}{}^4[x(1-u)^n, y(1-u)^n, z(1-u)^n, t(1-u)^n]$ du

=

$$\Lambda_1\Lambda_2 F_{13}{}^4 \begin{bmatrix} a_1, a_1, a_1, a_2, b_1, b_2, b_1, b_3, c_1, c_2, c_3, c_1; \Delta(n,\beta), \Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a), \Delta\left(\frac{n}{2}, \frac{\alpha+\beta-a}{2}\right), \Delta(\frac{n}{2}, \frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$$

………………………………………………….………..(1.2.13)

where

$F_{13}{}^4\ [a_1, a_1, a_1, a_2, b_1, b_2, b_1, b_3, c_1, c_2, c_3, c_1; x, y, z, t)$

$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q}(a_2)_r(a_3)_s(b_1)_{p+q+r+s}}{p!q!r!s!(c_1)_p(C_2)_q(C_3)_{r+s}} x^p y^q z^r t^s$

Then the probability density function of (1.2.13) is given by

F(U)=$\frac{u^{\alpha-1}(1-u)^{\beta-1}F_1(X_1)F_{13}{}^4[X_2]}{\Lambda_1\Lambda_2 F_{13}{}^4[X_{14}]}$

= 0 else where

Where

$$X_{14}=\begin{bmatrix} a_1,a_1,a_2,a_2,b_1,b_2,b_3,b_1,c_1,c_1,c_2,c_3;\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2});x;y;z;t \end{bmatrix}$$

=

$$\Lambda_1\Lambda_2 F_{14}{}^4\begin{bmatrix} a_1,a_1,a_2,a_2,b_1,b_2,b_1,b_3,c_1,c_2,c_1,c_3;\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2});x;y;z;t \end{bmatrix}$$

………………………………………………………..(1.2.14)

where

$F_{14}{}^4\ [a_1,a_1,a_2,a_2,b_1,b_2,b_1,b_3,c_1,c_2,c_1,c_3;x,y,z,t)$

$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q}\,(a_2)_{r+s}\,(b_1)_{p+r}(b_2)_q(b_3)_s}{p!q!r!s!(c_1)_{p+r}\,(C_2)_q\,(C_3)_s}\ x^p y^q z^r t^s$

Then the probability density function of (1.2.34) is given by

$F(U)=\frac{u^{\alpha-1}(1-u)^{\beta-1}F_1(X_1)F_{14}{}^4[X_2]}{\Lambda_1\Lambda_2 F_{14}{}^4[X_{15}]}$

= 0 else where

where

$$X_{15}=\begin{bmatrix} a_1,a_1,a_2,a_2,b_1,b_2,b_1,b_3,c_1,c_2,c_1,c_3;\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2});x;y;z;t \end{bmatrix}$$

[15] $\int_0^1 u^{\alpha-1}\ (1-u)^{\beta-1}\ F_1(a,b,1-a;\alpha,u;\frac{u}{2})$

$F_{15}{}^4[x(1-u)^n, y(1-u)^n, z(1-u)^n, t(1-u)^n]$ du

=

$$\Lambda_1\Lambda_2 F_{15}{}^4\begin{bmatrix} a_1,a_1,a_2,a_2,b_1,b_2,b_3,b_4,c_1,c_2,c_1,c_3;\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2});x;y;z;t \end{bmatrix}$$

………………………………………………….……..(1.2.15)

where

$F_{15}{}^4\ [a_1,a_1,a_2,a_2,b_1,b_2,b_3,b_4,c_1,c_2,c_1,c_3;x,y,z,t)$

$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_1)_{p+q}\,(a_2)_{r+s}\,(b_1)_p(b_2)_q(b_3)_r(b_4)_s}{p!q!r!s!(c_1)_{p+r}\,(C_2)_q\,(C_3)_s}\ x^p y^q z^r t^s$

Then the probability density function of (1.2.15) is given by

$F(U)=\frac{u^{\alpha-1}(1-u)^{\beta-1}F_1(X_1)F_{15}{}^4[X_2]}{\Lambda_1\Lambda_2 F_{15}{}^4[X_{16}]}$

= 0 else where

where

$$X_{16}=\begin{bmatrix} a_{1},, a_{1},, a_{2},, a_{2},b_{1},b_{2},, b_{3},b_{4},c_{1},c_{2},c_{1},c_{3;},\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2}); \text{x; y; z; t} \end{bmatrix}$$

[16] $\int_0^1 u^{\alpha-1}\ (1-u)^{\beta-1}\ F_1$(a,b,1-a;α,u;$\frac{u}{2}$)

${F_{16}}^4[x(1-u)^n, y(1-u)^n, z(1-u)^n, t(1-u)^n]$ du

=

$$\Lambda_1\Lambda_2{F_{16}}^4\begin{bmatrix} a_{1},, a_{1},, a_{2},, a_{3},b_{1},b_{2},, b_{1},b_{3},c_{1},c_{2},c_{3},c_{1;},\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2}); \text{x; y; z; t} \end{bmatrix}$$

……………………………………………………………..(1.2.16)

where

${F_{16}}^4\ [a_1, a_1, a_{2,} a_3, b_1, b_2, b_1, b_3, c_1, c_2, c_{3,} c_{1;} x, y, z, t)$

$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q}\,(a_{2,})_r\,(a_{3,})_s\,(b_{1,})_{p+r}(b_{2,})_q(b_{3,})_s}{p!q!r!s!(c_{1,})_{p+s}\ (c_{2,})_q\ (C_{3,})_r}\ x^p y^q z^r t^s$

Then the probability density function of (1.2.16) is given by

F(U)=$\frac{u^{\alpha-1}(1-u)^{\beta-1}F_1(X_1){F_{16}}^4[X_2]}{\Lambda_1\Lambda_2{F_{16}}^4[X_{17}]}$

= 0 elsewhere

where

$$X_{17}=\begin{bmatrix} a_{1},, a_{1},, a_{2},, a_{3},b_{1},b_{2},, b_{1},b_{3},c_{1},c_{2},c_{3},c_{1;},\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2}); \text{x; y; z; t} \end{bmatrix}$$

[17] $\int_0^1 u^{\alpha-1}\ (1-u)^{\beta-1}\ F_1$(a,b,1-a;α,u;$\frac{u}{2}$)

${F_{17}}^4[x(1-u)^n, y(1-u)^n, z(1-u)^n, t(1-u)^n]$ du

=

$$\Lambda_1\Lambda_2{F_{17}}^4\begin{bmatrix} a_{1},, a_{1},, a_{2},, a_{3},b_{1},b_{2},, b_{1},b_{3},c_{1},c_{2},c_{3},c_{3;},\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2}); \text{x; y; z; t} \end{bmatrix}$$

……………………………………………………………..(1.2.17)

where

${F_{17}}^4\ [a_1, a_1, a_{2,} a_3, b_1, b_2, b_1, b_3, c_1, c_2, c_{3,} c_{3;} x, y, z, t)$

$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_{p+q}\,(a_{2,})_r\,(a_{3,})_s\,(b_{1,})_{p+r}(b_{2,})_q(b_{3,})_s}{p!q!r!s!(c_{1,})_{p}\ (c_{2,})_q\ (C_{3,})_{r+s}}\ x^p y^q z^r t^s$

Then the probability density function of (1.2.17) is given by

F(U)=$\frac{u^{\alpha-1}(1-u)^{\beta-1}F_1(X_1){F_{17}}^4[X_2]}{\Lambda_1\Lambda_2{F_{17}}^4[X_{18}]}$

= 0 elsewhere

where

$$X_{18}=\begin{bmatrix} a_{1},a_{1},a_{2},a_{3},b_{1},b_{2},b_{1},b_{3},c_{1},c_{2},c_{3},c_{3;},\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2});x;y;z;t \end{bmatrix}$$

[18] $\int_0^1 u^{\alpha-1}\,(1-u)^{\beta-1}\,F_1(\text{a,b,1-a};\alpha,\text{u};\frac{u}{2})$

$F_{18}{}^{4}[x(1-u)^n,y(1-u)^n,z(1-u)^n,t(1-u)^n]$ du

=

$$\Lambda_1\Lambda_2 F_{18}{}^{4}\begin{bmatrix} a_{1},a_{1},a_{2},a_{2},b_{1},b_{1},b_{1},b_{1},c_{1},c_{2},c_{1},c_{2;},\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2});x;y;z;t \end{bmatrix}$$

…………………………………………………….………..(1.2.18)

where

$F_{18}{}^{4}\,[a_1,a_1,a_{2,}a_2,b_1,b_1,b_1,b_1,c_1,c_2,c_{1,}c_{2;}x,y,z,t)$

$=\sum_{p,q,r,s=0}^{\infty}\frac{(a_{1})_{p+q}\,(a_{2})_{r}\,(a_{3})_{s}\,(b_{1})_{p+r}(b_{2})_{q}(b_{3})_{s}}{p!q!r!s!(c_{1})_{p}\,(c_{2})_{q}\,(C_{3})_{r+s}}\,x^p y^q z^r t^s$

Then the probability density function of (1.2.18) is given by

F(U)=$\frac{u^{\alpha-1}(1-u)^{\beta-1}F_1(X_1)F_{18}{}^{4}[X_2]}{\Lambda_1\Lambda_2 F_{18}{}^{4}[X_{19}]}$

= 0 elsewhere

where

$$X_{19}=\begin{bmatrix} a_{1},a_{1},a_{2},a_{2},b_{1},b_{1},b_{1},b_{1},c_{1},c_{2},c_{1},c_{2;},\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2});x;y;z;t \end{bmatrix}$$

[19] $\int_0^1 u^{\alpha-1}\,(1-u)^{\beta-1}\,F_1(\text{a,b,1-a};\alpha,\text{u};\frac{u}{2})$

$F_{19}{}^{4}[x(1-u)^n,y(1-u)^n,z(1-u)^n,t(1-u)^n]$ du

=

$$\Lambda_1\Lambda_2 F_{19}{}^{4}\begin{bmatrix} a_{1},a_{1},a_{2},a_{3},b_{1},b_{1},b_{1},b_{1},c_{1},c_{2},c_{1},c_{2;},\Delta(n,\beta),\Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a),\Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right),\Delta(\frac{n}{2},\frac{1-b+\alpha+\beta-a}{2});x;y;z;t \end{bmatrix}$$

………………………………………………………..(1.2.19)

where

$F_{19}{}^{4}\,[a_1,a_1,a_{2,}a_3,b_1,b_1,b_1,b_1,c_1,c_2,c_{1,}c_{2;}x,y,z,t)$

$=\sum_{p,q,r,s=0}^{\infty}\frac{(a_{1})_{p+q}\,(a_{2})_{r}\,(a_{3})_{s}\,(b_{1})_{p+r+r+s}}{p!q!r!s!(c_{1})_{p+r}\,(c_{2})_{q+s}}\,x^p y^q z^r t^s$

Then the probability density function of (1.2.19) is given by

F(U)=$\frac{u^{\alpha-1}(1-u)^{\beta-1}F_1(X_1)F_{19}{}^{4}[X_2]}{\Lambda_1\Lambda_2 F_{19}{}^{4}[X_{20}]}$

= 0 elsewhere

where

$$X_{20}=\begin{bmatrix} a_{1,}, a_{1,}, a_{2,}, a_{3,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{2,} c_{1,} c_{2;,} \Delta(n,\beta), \Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a), \Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right), \Delta(\frac{n}{2}, \frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$$

[20] $\int_0^1 u^{\alpha-1}\ (1-u)^{\beta-1}\ F_1$(a,b,1-a;α,u;$\frac{u}{2}$)

${F_{20}}^4[x(1-u)^n, y(1-u)^n, z(1-u)^n, t(1-u)^n]$ du

=

$$\Lambda_1\Lambda_2 {F_{20}}^4 \begin{bmatrix} a_{1,}, a_{1,}, a_{3,}, a_{4,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{1,} c_{2,} c_{2;,} \Delta(n,\beta), \Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a), \Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right), \Delta(\frac{n}{2}, \frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$$

…………………………………………………………..(1.2.20)

where

${F_{20}}^4\ [a_1, a_2, a_{3,} a_4, b_1, b_1, b_1, b_1, c_1, c_1, c_{2,} c_{2;} x, y, z, t)$

$= \sum_{p,q,r,s=0}^{\infty} \frac{(a_{1,})_p\ (a_{2,})_q\ (a_{3,})_r\ (a_{4,})_s (b_{1,})_{p+q+r+s}}{p!q!r!s!(c_{1,})_p\ \ (C_{2,})_q\ (C_{3,})_{r+s}}\ x^p y^q z^r t^s$

Then the probability density function of (1.2.20) is given by

F(U)=$\frac{u^{\alpha-1}(1-u)^{\beta-1} F_1(X_1) {F_{20}}^4[X_2]}{\Lambda_1\Lambda_2 {F_{20}}^4[X_{21}]}$

= 0 elsewhere

where

$$X_{21}=\begin{bmatrix} a_{1,}, a_{2,}, a_{3,}, a_{4,} b_{1,} b_{1,}, b_{1,} b_{1,} c_{1,} c_{1,} c_{2,} c_{2;,} \Delta(n,\beta), \Delta(n,\alpha+\beta-a-b) \\ \Delta(n,\alpha+\beta-a), \Delta\left(\frac{n}{2},\frac{\alpha+\beta-a}{2}\right), \Delta(\frac{n}{2}, \frac{1-b+\alpha+\beta-a}{2}); x; y; z; t \end{bmatrix}$$

Conclusion

In this paper, we specialized parameters and argument, Hypergeometric function F_E $(\alpha_1, \alpha_1, \alpha_1, \beta_1, \beta_2, \beta_2; \gamma_1, \gamma_2, \gamma_3;$ sinhx, sinhy, sinhz) F_G , F_k and F_N can be reduced to the hypergeometric function of Bailey's F_4 (α_1 , β_2 , γ_2, γ_3; - sinhy, - sinh z) and also discussed their reducible cases into Horn's function . In the present paper we consider hypergeometric function of three variables and obtain its interesting reducible case into Bailey's F_4 & Horn's function.

Bibliography

1. SETHI P. L. AND SIDHU B, S. (2005) : Multivariate integral representation suggested by Laguerre and Jacobi polynomials of matrix argument Vihnana Parshad Anusandhan Patrika, vol, 48, No. 2, p.p 171-219.

2. SHEKHWAT A.S.(2005): Fredholm type integral equations and certain polynomials " published in vol. 45, No. 4, 471-480, kyungpook Mathematical Journal kyungpook national university, Taegu, Korea.

3. SHEKHWAT A.S. (2005): Some Integral Properties of general Class of polynomials associated with feynman integrals " published in Vol. 28, No. 2 , 183-189, Bulletin of Malaysian Mathematical Science Society, University of Malaysia, Pinang, Malaysia.

4. SHEKHWAT A.S. (2007): " An application of Fractional Derivatives of certain special functins" published in Vol.33, No.2 ,325-334,Soochow Journal of Mathematics, Soochow University ,Taipei, Taiwan, Republic of China.

5. SHEKHWAT A.S. (2007): Paper entitled "Fractional Derivative Associated with the multivariable Polynomials "published in Vol.4, No.4 , 495-500, Kyungpook Mathematical Journal, Kyunpook National University, Taegu, Korea.

6. SHEKHWAT A.S. (2005): Paper entitled "Fractional integration pertaining to a product of certain Transcendental functions" published in Vol.35, 47-58, Jnanabha, The Vijnana Parishad of India, Bundelkhand University, U.P., India.

7. SHEKHWAT A.S. (2010): Paper entitled "Fractional integration of certain Transcendental functions" published in Vol.24, No. 1 , 11-20, Ganita Sandesh, Rajasthan Ganita Parishad, Ajmer.

8. SHEKHWAT A.S. (2012): Paper entitled "Fractional integration associated with the Transcendental functions" published in Vol.12, Issue 4 version 1.0 , Global Journal of Science Frontier Research, U.S.A.

9. SHEKHWAT A.S. (2012): Paper entitled " An Integral associated with H- function of several complex variables" accepted in Vol. 12, Issue 13, version 1.0 , Global Journal of Science Frontier Research, U.S.A.

10.SHEKHWAT A. S. (2005): Paper entitled " An integral involving general polynomials and the H-function of several complex variables " published in Vol.36, No.3 ,255-260, Tamkang journal of mathematics, Tamkang University, Taiwan, Republic of China.

11.UPANDHYAYA LALIT MOHAN AND DHAMI H. S. (2001); Matrix Generalizations of Multiple Hypergeometric Function, 1818 IMA Preprint Series, University of Minnesota, Minneapolis, U.S.A.

12.UPADHYAYA LALIT MOHANAND DHAMI H. S. (2002): Appell's and Humbert's function of matrix arguments-I 1848 IMA Preprints Series, University of Minnesota, Minneapolis, U.S.A.

13.VYAS, MAHESH AND SETHI P. L. (2001): Four variables associated with hypergeometric functions Ph. D Thesis , J.N.V. university jodhpur .